

Bulk viscosity of spin-one color superconducting strange quark matter

Xinyang Wang¹ and Igor A. Shovkovy^{1,2,*}

¹*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*

²*Department of Applied Sciences and Mathematics,
Arizona State University, Mesa, Arizona 85212, USA*

(Dated: September 2, 2010)

The bulk viscosity in spin-one color-superconducting strange quark matter is calculated by taking into account the interplay between the nonleptonic and semi-leptonic weak processes. In agreement with previous studies, it is found that the inclusion of the semi-leptonic processes may result in non-negligible corrections to the bulk viscosity in a narrow window of temperatures. The effect is generally more pronounced for pulsars with longer periods. Compared to the normal phase, however, this effect due to the semi-leptonic processes is less pronounced in spin-one color superconductors. Assuming that the critical temperature of the phase transition is much larger than 40 keV, the main effect of spin-one color superconductivity in a wide range of temperatures is an overall increase of the bulk viscosity with respect to the normal phase. The corresponding enhancement factor reaches up to about 9 in the polar and *A*-phases, about 25 in the planar phase and about 29 in the CSL phase. This factor is determined by the suppression of the nonleptonic rate in color-superconducting matter and, therefore, may be even larger if all quark quasiparticles happen to be gapped.

PACS numbers: 12.38.Mh, 12.38.Aw, 12.15.Ji, 26.60.Dd

I. INTRODUCTION

Understanding the physical properties of baryonic matter above nuclear saturation density is one of the fundamental challenges in modern nuclear astrophysics. Many aspects of neutron stars (e.g., the mass-radius relation, cooling and rotational dynamics, glitches and pulsar kicks) depend on these properties. For example, the equation of state of supranuclear baryonic matter plays the key role in determining the maximum possible mass of neutron stars. The harder (softer) equation of state is, the larger (smaller) maximum mass can be. The equation of state of dense baryonic matter is also one of the essential ingredients that determines the dynamics of core-collapse supernovae and, in turn, the mass distribution of black holes in the Universe [1]. Several characteristics of the gravitational wave emission from merging neutron stars [2], which may be observed, e.g., by advanced Laser Interferometer Gravitational Wave Observatory (LIGO), are sensitive to the details of the equation of state.

From the theoretical viewpoint, currently there is no consensus even regarding the qualitative state of matter at the highest densities reached in stellar cores. The most conservative possibility is that such matter is made of only nucleonic degrees of freedom. The study of several neutron stars in Ref. [3], for example, does not exclude such a possibility, although a phase transition may be in agreement with their analysis, provided no extreme softening of the equation of state occurs. Another astrophysical determination of masses and radii of three neutron stars in Ref. [4] suggests, however, that the actual equation of state is too soft to be pure nucleonic. Such

contradictory interpretations are representative and show that the current knowledge is too limited to settle the issue. Theoretically, it may be also appropriate to mention that the observables associated with the equation of state alone have limited power to probe the actual nature of dense matter [5, 6]. In fact, a true insight regarding the stellar interior may require a comprehensive understanding not only of the thermodynamical, but also transport properties and neutrino emission rates of various possible states of superdense matter.

In this paper, we assume that baryonic matter at the highest stellar densities is deconfined quark matter. The possible formation of quark matter in stars is an old hypothesis [7] that dates back to the time when the concept of quarks was first introduced [8, 9]. This is also supported by general considerations [10] based on the property of asymptotic freedom in quantum chromodynamics (QCD) [11]. The main uncertainties of this scenario are (i) the value of the critical density, at which the deconfinement transition occurs, and (ii) the actual highest density reached in stars. If quark matter is formed, as we assume here, it is also likely to be a color-superconductor [12, 13]. (For reviews on color superconductivity and its general effects on stellar properties see, for example, Refs. [14–19].) In this paper, in particular, we concentrate on the scenario, in which color superconductivity is due to same-flavor, spin-one Cooper pairing [20–23].

The fact of liberation of quark degrees of freedom and the formation of a color-superconducting state of matter is likely to be revealed through a detailed study of the observational features of neutron stars. For example, one promising class of observables is related to the rates of weak processes. Such processes are known to be responsible for the cooling rates [24] and damping of the rotational (r-mode) instabilities [25] in stars. The cooling is primarily determined by the neutrino emission

*Electronic address: igor.shovkovy@asu.edu

rate, while the damping of r-modes is controlled by the viscosity of dense matter [26].

The bulk viscosity in the normal phase of three-flavor quark matter is usually dominated by the nonleptonic weak processes [27–33]. It was argued in Ref. [34], however, that the interplay between the semi-leptonic and nonleptonic processes may be rather involved even in the normal phase of quark matter. Indeed, because of the resonance-like dynamics responsible for the bulk viscosity and because of a subtle interference between the two types of the weak processes, a larger rate of the nonleptonic processes may not automatically mean its dominant role. In fact, it was shown that the contributions of the two types of weak processes are generally not separable and that, for a range of parameters, taking into account the semi-leptonic processes may substantially modify the nonleptonic result [34].

In this paper, we extend the analysis of Ref. [34] and study the effect that spin-one color superconductivity has on the bulk viscosity when the interplay between the two types of weak processes is carefully taken into account. (For calculation of the bulk viscosity in other color superconducting phases see Refs. [35–39].) The necessary ingredients for the calculation of the bulk viscosity are the rates of semi-leptonic (Urca) and nonleptonic weak processes. While the needed rates for the semi-leptonic processes in several spin-one color superconducting phases were obtained several years ago in Ref. [37], the corresponding rates of the nonleptonic processes remained unknown until very recently [40]. Here we utilize both to obtain the bulk viscosity.

The rest of the paper is organized as follows. The general formalism for the calculation of the bulk viscosity in strange quark matter with several active weak processes is reviewed in Sec. II. This formalism is then used in Sec. III to obtain our main results for the bulk viscosity as a function of temperature and the frequency of density oscillations. There we also study the enhancement effect of color superconductivity on the bulk viscosity, as well as the interplay of semi-leptonic and nonleptonic processes. In Sec. IV, we discuss the results and their potential implications for the physics of compact stars. Two appendices at the end of the paper contain our fits for the numerical suppression factors of the semi-leptonic and nonleptonic rates in spin-one color superconducting strange quark matter.

II. FORMALISM

In this study, in order to calculate the bulk viscosity in the presence of several types of active weak processes, we follow the general formalism of Ref. [34]. We assume that small oscillations of the quark matter density are described by $\delta n = \delta n_0 \text{Re}(e^{i\omega t})$ where δn_0 is the magnitude of the oscillations. For such a periodic process, the bulk viscosity ζ is defined as the coefficient in the expression for the energy-density dissipation averaged over one

period, $\tau = 2\pi/\omega$,

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2, \quad (1)$$

where \vec{v} is the hydrodynamic velocity associated with the density oscillations. By making use of the continuity equation, $\dot{n} + n \nabla \cdot \vec{v} = 0$, we derive

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\frac{\zeta \omega^2}{2} \left(\frac{\delta n_0}{n} \right)^2. \quad (2)$$

Such an energy-density dissipation of a pulsating hydrodynamic flow is the outcome of a net work done on a macroscopic volume over a period of the oscillation,

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = \frac{n}{\tau} \int_0^\tau P \dot{\mathcal{V}} dt, \quad (3)$$

where $\mathcal{V} \equiv 1/n$ is the specific volume. By matching the hydrodynamic definition in Eq. (2) with the relation in Eq. (3), we derive the expression for the bulk viscosity,

$$\zeta = -\frac{2}{\omega^2} \left(\frac{n}{\delta n_0} \right)^2 \frac{n}{\tau} \int_0^\tau P \dot{\mathcal{V}} dt. \quad (4)$$

The dominant mechanism behind the bulk viscosity is related to weak processes [26–29]. A periodic oscillation of the density is responsible for an instantaneous departure from β -equilibrium in the system. As a result, the forward and backward weak processes (e.g., $u + d \rightarrow s + u$ and $s + u \rightarrow u + d$), which have equal rates in equilibrium, become unbalanced. Their net effect is to restore the equilibrium composition. However, since the weak rates are relatively slow, a substantial time lag between the oscillations of the fermion number density (and, thus, the specific volume) and the chemical composition (and, thus, the pressure) develops. If the resulting relative phase shift of the two oscillations is $\Delta\phi$, one finds from Eqs. (3) and (4) that the corresponding energy dissipation and the bulk viscosity are proportional to $\sin \Delta\phi$. (Note that the departure from the thermal equilibrium is negligible because it is restored by strong forces on much shorter time scales.)

It should be clear that the instantaneous flavor composition in oscillating quark matter and the rate difference of the forward and backward weak processes in Fig. 1 are related to each other. The difference of the rates changes the composition, while the composition in turn influences the difference of rates. The corresponding dynamics can be conveniently described in terms of the time dependent deviations of the chemical potentials from their equilibrium values.

In β equilibrium, the chemical potentials of the three lightest quarks are related as follows: $\mu_s = \mu_d = \mu_u + \mu_e$. Here μ_u , μ_d and μ_s are the chemical potentials of up, down and strange quarks, while μ_e is the electron chemical potential. In pulsating matter, the instantaneous

departure from equilibrium is described by the following two independent parameters:

$$\delta\mu_1 \equiv \mu_s - \mu_d = \delta\mu_s - \delta\mu_d, \quad (5a)$$

$$\delta\mu_2 \equiv \mu_s - \mu_u - \mu_e = \delta\mu_s - \delta\mu_u - \delta\mu_e, \quad (5b)$$

where $\delta\mu_i$ denotes the deviation of the chemical potential μ_i from its equilibrium value. (Note that $\delta\mu_3 \equiv \mu_d - \mu_u - \mu_e = \delta\mu_2 - \delta\mu_1$ is not independent.) When $\delta\mu_i$ are non-zero, the corresponding pairs of forward and backward weak processes in Fig. 1 have different rates. To leading order, the rate differences are linear in $\delta\mu_i$,

$$\Gamma_{(a)} - \Gamma_{(b)} = -\lambda_1 \delta\mu_1, \quad (6a)$$

$$\Gamma_{(c)} - \Gamma_{(d)} = -\lambda_2 \delta\mu_2, \quad (6b)$$

$$\Gamma_{(e)} - \Gamma_{(f)} = -\lambda_3 (\delta\mu_2 - \delta\mu_1). \quad (6c)$$

The corresponding λ -rates have been calculated for the normal phase [27, 28, 41, 42] as well as several color superconducting phases of quark matter [36, 37, 40]. The results for the normal phase, in particular, read

$$\lambda_1^{(0)} \simeq \frac{64}{5\pi^3} G_F^2 \cos^2 \theta_C \sin^2 \theta_C \mu_d^5 T^2, \quad (7a)$$

$$\lambda_2^{(0)} \simeq \frac{17}{40\pi} G_F^2 \sin^2 \theta_C \mu_s m_s^2 T^4, \quad (7b)$$

$$\lambda_3^{(0)} \simeq \frac{17}{15\pi^2} G_F^2 \cos^2 \theta_C \alpha_s \mu_d \mu_u \mu_e T^4. \quad (7c)$$

These will be used below as a benchmark for the rates in spin-one color-superconducting phases.

The semi-leptonic rate λ_3 is determined by the Urca processes $u + e^- \rightarrow d + \nu_e$ and $d \rightarrow u + e^- + \bar{\nu}_e$, shown in diagrams (e) and (f) in Fig. 1. It was calculated in Ref. [37] for four different spin-one color-superconducting phases of quark matter. The result has a form of the product of the rate in the normal phase $\lambda_3^{(0)}$ and a phase-specific suppression factor,

$$\lambda_3 = \lambda_3^{(0)} \left[\frac{1}{3} + \frac{2}{3} H \left(\frac{\phi}{T} \right) \right], \quad (8)$$

where ϕ is the spin-one color-superconducting gap parameter, and $H(\phi/T)$ is a suppression factor for the processes involving gapped quasiparticles. (The first term in square brackets is the contribution of ungapped quasiparticles.) When $\phi \rightarrow 0$, the suppression factor $H(\phi/T)$ approaches 1 and the normal phase result is restored. A simple fit to the numerical data of Ref. [37] for $H(\phi/T)$ is presented in Appendix A.

Because of similar kinematics and phase space constraints for the other pair of semi-leptonic processes, $u + e^- \rightarrow s + \nu_e$ and $s \rightarrow u + e^- + \bar{\nu}_e$, shown in diagrams (c) and (d) in Fig. 1, the dependence of the rate λ_2 on the color-superconducting gap should take the same form as λ_3 in Eq. (8), i.e.,

$$\lambda_2 = \lambda_2^{(0)} \left[\frac{1}{3} + \frac{2}{3} H \left(\frac{\phi}{T} \right) \right]. \quad (9)$$

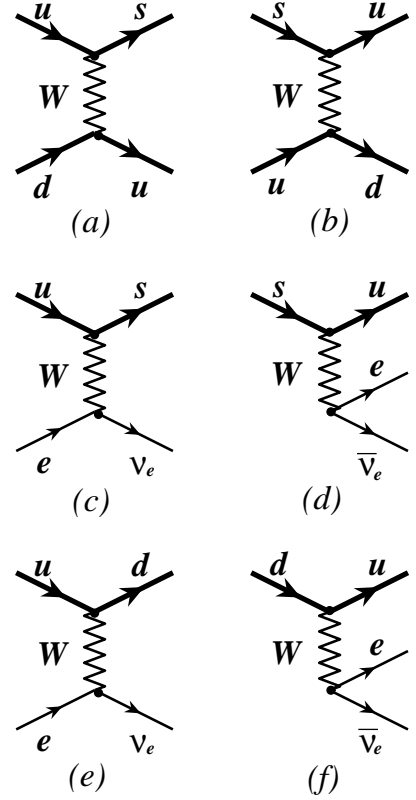


FIG. 1: Diagrammatic representation of the β processes that contribute to the bulk viscosity of dense quark matter.

In contrast, the rate λ_1 is determined by the nonleptonic processes $u + d \rightarrow s + u$ and $s + u \rightarrow u + d$, see diagrams (a) and (b) in Fig. 1, which have a qualitatively different kinematics. In spin-one color-superconducting phases of quark matter, this was recently calculated in Ref. [40]. The numerical result can be conveniently summarized by the following expression:

$$\lambda_1 = \lambda_1^{(0)} \left[\mathcal{N} + (1 - \mathcal{N}) \tilde{H} \left(\frac{\phi}{T} \right) \right], \quad (10)$$

where, in addition to the suppression factor $\tilde{H}(\phi/T)$, we also introduced a constant \mathcal{N} , which determines a relative contribution of the ungapped quasiparticles to the corresponding rate. In the four spin-one phases studied in Ref. [40], the constant takes the following values: $\mathcal{N}^A = \mathcal{N}^{\text{polar}} = 1/9$, $\mathcal{N}^{\text{planar}} \approx 0.0393$, and $\mathcal{N}^{\text{CSL}} = 928/27027 \approx 0.0343$. A simple fit to the numerical data for $\tilde{H}(\phi/T)$ is given in Appendix B.

When the rates (8), (9) and (10) are known, the calculation of the instantaneous pressure and, thus, the bulk viscosity from Eq. (4) is straightforward [34]. Here we quote only the final expression for the viscosity,

$$\zeta = \zeta_1 + \zeta_2 + \zeta_3, \quad (11)$$

where

$$\zeta_1 = \frac{n}{\omega} \frac{\alpha_2 \alpha_3}{g_1^2 + g_2^2} \left[\alpha_1 \alpha_2 \alpha_3 C_1^2 + (\alpha_1 + \alpha_2 + \alpha_3) (A_1 C_2 - A_2 C_1)^2 \right], \quad (12a)$$

$$\zeta_2 = \frac{n}{\omega} \frac{\alpha_1 \alpha_3}{g_1^2 + g_2^2} \left[\alpha_1 \alpha_2 \alpha_3 C_2^2 + (\alpha_1 + \alpha_2 + \alpha_3) [(A_2 - B_2) C_1 - A_2 C_2]^2 \right] \quad (12b)$$

$$\zeta_3 = \frac{n}{\omega} \frac{\alpha_1 \alpha_2}{g_1^2 + g_2^2} \left[\alpha_1 \alpha_2 \alpha_3 (C_1 - C_2)^2 + (\alpha_1 + \alpha_2 + \alpha_3) (B_1 C_2 - B_2 C_1)^2 \right], \quad (12c)$$

and

$$g_1 = -\alpha_1 \alpha_2 \alpha_3 + (\alpha_1 + \alpha_2 + \alpha_3) (B_1 A_2 - A_1 B_2), \quad (13a)$$

$$g_2 = \alpha_1 \alpha_2 (B_1 - B_2) + \alpha_1 \alpha_3 (A_2 - B_2) + \alpha_2 \alpha_3 A_1. \quad (13b)$$

Here $\alpha_i \equiv n\omega/\lambda_i$ ($i = 1, 2$) and n is the baryon density of quark matter. The quantities A_i , B_i and C_i are susceptibility-like functions, see Ref. [34] for the definition. To leading order in ϕ/μ_i , they are the same as in the normal phase.

For comparison, let us also note that the bulk viscosity in the limit of the vanishing semi-leptonic rates reads

$$\zeta_{\text{non}} = \frac{n}{\omega} \frac{\alpha_1 C_1^2}{\alpha_1^2 + A_1^2}. \quad (14)$$

III. NUMERICAL RESULTS FOR BULK VISCOSITY

In our calculation of the bulk viscosity in spin-one color-superconducting quark matter below, we choose the same two representative sets of model parameters as in Ref. [34]:

Set A	Set B
$n = 5\rho_0$	$n = 10\rho_0$
$m_s = 300 \text{ MeV}$	$m_s = 140 \text{ MeV}$
$\alpha_s = 0.2$	$\alpha_s = 0.1$

In both cases, the masses of light quarks are the same: $m_u = 5 \text{ MeV}$ and $m_d = 9 \text{ MeV}$. In accordance with general expectations, the values of the strange quark mass m_s and the strong coupling constant α_s should be larger (smaller) in the case of lower (higher) density. This qualitative property is reflected in the model parameters in Set A (Set B). The values of all chemical potentials as well as the coefficient functions A_i , B_i and C_i for each set of parameters are quoted in Table I.

It may be appropriate to briefly comment about the choice of the strong coupling constant α_s in the model at hand. The values of α_s in both sets of parameters may seem abnormally small. Indeed, the running coupling in

QCD is about 0.12 at the scale of M_Z (mass of Z boson) and about 0.32 at $\sqrt{3} \text{ GeV}$ [43]. However, here we use the model parameter α_s only in order to capture several qualitative (Fermi liquid) effects in quark matter. Its nonzero value allows (i) to avoid the underestimation of the rate of semi-leptonic processes due to a limited phase space [24] and (ii) to mimic the modification of the quark equation of state due to strong interactions, see Ref. [34] for details. The naive extension of the corresponding leading order corrections to the regime of strong coupling is problematic. Not only this would imply the use of the perturbative results beyond the range of their validity, but this would also lead to very large and seemingly unphysical effects on the equation of state, used to determine the susceptibility functions A_i , B_i and C_i . (Notably, if the equation of state is kept unchanged, the increase of α_s in the $\lambda_3^{(0)}$ -rate, even by an order of magnitude, has little effect on the viscosity.) This dilemma could be resolved by properly accounting the non-perturbative dynamics of QCD. At present, however, such a task seems insurmountable at the low energy scales relevant for neutron stars. For the purposes of this study, therefore, we treat α_s as a small independent parameter that captures only some qualitative properties of quark matter.

Here the critical temperature of the spin-one color-superconducting phase transition is assumed to be $T_c = 2 \text{ MeV}$. This may be a somewhat high, but still reasonable value for T_c . Indeed, in QCD the spin-one gap is estimated to be about two orders of magnitude smaller than the spin-zero gap [20–22], and the latter is naturally of order 100 MeV [14–19]. Even higher values of the spin-one gap have been reported in Ref. [44]. The effect of varying the critical temperature is easy to understand and will be briefly discussed below. As in Ref. [37], we use the following model temperature dependence of the gap parameter:

$$\phi(T) = \phi_0 \sqrt{1 - \left(\frac{T}{T_c}\right)^2}, \quad \text{for } T < T_c \quad (15)$$

with ϕ_0 being the value of the gap parameter at $T = 0$. Note that the ratio T_c/ϕ_0 depends on the choice of the phase [22]. The approximate values of this ratio are 0.8 (CSL), 0.66 (planar), 0.49 (polar), and 0.81 (A -phase).

For model parameters in Set A, the numerical results are presented in Fig. 2. As we can see, the value of T_c determines the point where the bulk viscosity starts to deviate from the benchmark result in the normal phase (shown by the gray solid line). The upper panels show the dependence of the bulk viscosity ζ on temperature for two representative values of the oscillation frequency, $\tau^{-1} = 10 \text{ Hz}$ and $\tau^{-1} = 1000 \text{ Hz}$. The lower panels in the same figure show the temperature dependence of the ratio ζ/ζ_{non} , where ζ is the bulk viscosity that takes into account all weak processes, while ζ_{non} is an approximate result, see Eq. (14), in which only the nonleptonic processes are included and the semi-leptonic processes are not. When the ratio ζ/ζ_{non} is substantially larger than

TABLE I: Two sets of parameters used in the calculation of the bulk viscosity.

model	μ_e [MeV]	μ_u [MeV]	$\mu_d = \mu_s$ [MeV]	A_1 [MeV]	A_2 [MeV]	B_1 [MeV]	B_2 [MeV]	C_1 [MeV]	C_2 [MeV]
Set A	39.139	402.463	441.602	239.432	127.937	111.386	-3.726×10^4	-60.463	-60.460
Set B	7.396	495.275	502.671	324.556	164.288	160.268	-2.080×10^6	-10.692	-10.709

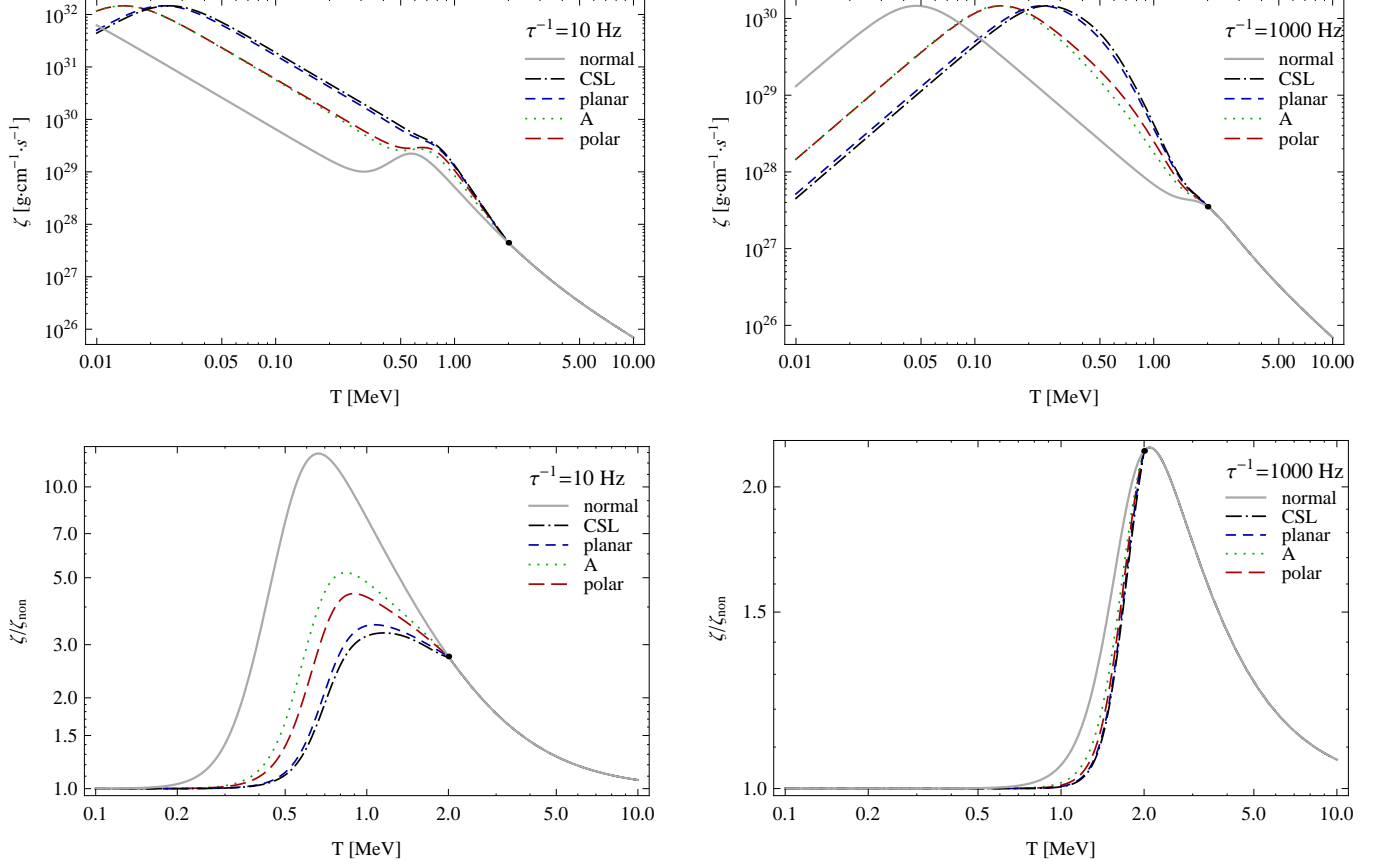


FIG. 2: (color online). Temperature dependence of bulk viscosity ζ and the ratio ζ/ζ_{non} for model parameters in Set A and the spin-one color-superconducting critical temperature $T_c = 2$ MeV. The results for two fixed frequencies of the density oscillations are shown.

1, it is an indication that the semi-leptonic processes play an important role and, thus, cannot be neglected.

Compared to the normal phase result, the main features of the temperature dependences in spin-one color-superconducting phases (see the upper panels in Fig. 2) are (i) a smoothed shape of the semi-leptonic “hump” and (ii) an overall enhancement of the bulk viscosity due to color superconductivity for a substantial range of temperatures below T_c .

As in the case of the normal phase, the semi-leptonic processes are responsible for an increase (“hump”) of the bulk viscosity in a region of temperatures around T_{hump} ,

where

$$T_{\text{hump}}^{(\text{Set A})} \simeq 2.1 \text{ MeV} \left(\frac{1 \text{ ms}}{\tau} \right)^{1/4}, \quad (16a)$$

$$T_{\text{hump}}^{(\text{Set B})} \simeq 1.4 \text{ MeV} \left(\frac{1 \text{ ms}}{\tau} \right)^{1/4} \quad (16b)$$

are the approximate positions of the peak of the hump in the normal phase in the case of the model parameters in Set A and Set B, respectively. In order to derive these results, we used an approximate expression for the bulk viscosity in Eq. (24) of Ref. [34], which is valid when the nonleptonic rate is infinitely large while the semi-leptonic rates are finite. The maximum of that expression corresponds to $\lambda_2 + \lambda_3 = n\omega A_1/(B_1 A_2 - B_2 A_1)$, whose solution determines an approximate value for T_{hump} . Two

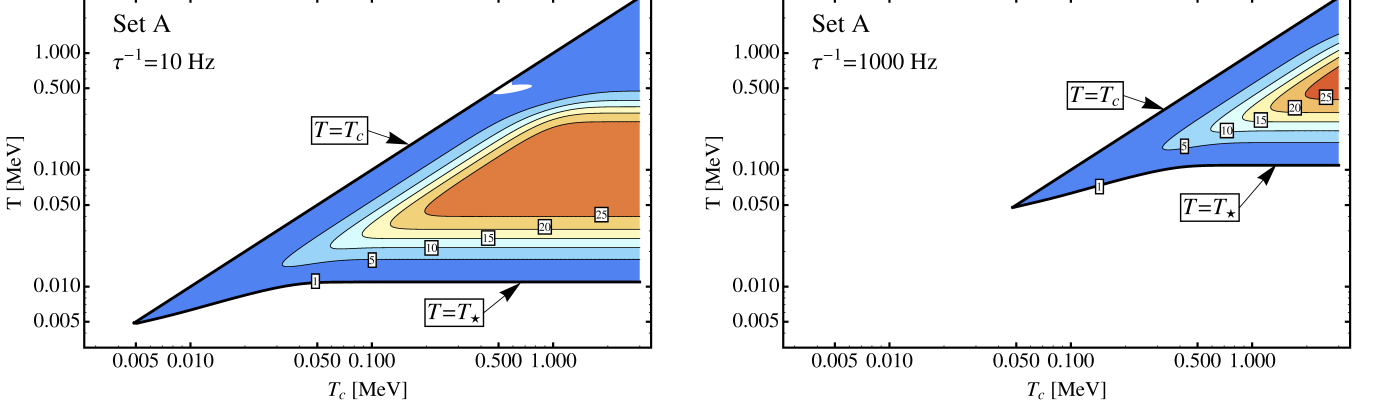


FIG. 3: (color online). The contour plot of the bulk viscosity enhancement factor due to spin-one color superconductivity. The results are for the CSL phase in a model with the parameters in Set A. The results for two frequencies of the density oscillations are shown: $\tau^{-1} = 10$ Hz (left panel) and $\tau^{-1} = 1000$ Hz (right panel). The ratio $\zeta_{\text{CSL}}/\zeta_{\text{normal}}$ is larger than 1 in the colored regions and is equal to or less than 1 in the white region. The contours are labeled by the corresponding values of the ratio $\zeta_{\text{CSL}}/\zeta_{\text{normal}}$.

remarks are in order here: (i) the scaling law $T_{\text{hump}} \propto 1/\tau^{1/4}$ follows from the power-law temperature dependence of the semi-leptonic rates $\lambda_2, \lambda_3 \propto T^4$ and (ii) the overall value in Eq. (16) is slightly corrected to match the actual numerical results in the case of a finite nonleptonic rate.

When $T_c \gtrsim T_{\text{hump}}$ the semi-leptonic hump is partially washed out by the presence of color superconductivity. This is most clearly seen from the ratio of the bulk viscosities ζ/ζ_{non} in the lower panels in Fig. 2. While the inclusion of the semi-leptonic processes leads to an increase of the viscosity, the effect is not as large as in the normal phase. Of course, this conclusion is sensitive to the choice of the color-superconducting critical temperature T_c . In general, two qualitatively different regimes can be realized. When $T_c \lesssim T_{\text{hump}}$, the hump occurs in the normal phase and, therefore, its shape is almost unaffected by color superconductivity. In the opposite case, $T_c \gtrsim T_{\text{hump}}$, the effect is present and gets stronger as T_c increases relative to T_{hump} .

Now, let us turn to an overall enhancement of the bulk viscosity due to color superconductivity below T_c . This is observed almost for the whole range of temperatures $T_{0,\text{max}} \lesssim T \leq T_c$, where $T_{0,\text{max}}$ is the temperature at which the bulk viscosity of the normal phase has a global maximum. The value of $T_{0,\text{max}}$ can be easily estimated by considering an approximate expression for the bulk viscosity (14) when only the nonleptonic processes are taken into account. The maximum of Eq. (14) corresponds to $\alpha_1 = A_1$. After solving this for the temperature, we obtain

$$T_{0,\text{max}}^{(\text{Set A})} \simeq 47 \text{ keV} \sqrt{\frac{1 \text{ ms}}{\tau}}, \quad (17a)$$

$$T_{0,\text{max}}^{(\text{Set B})} \simeq 41 \text{ keV} \sqrt{\frac{1 \text{ ms}}{\tau}}, \quad (17b)$$

where τ is the period of oscillations measured in milliseconds. Notably, the location of the maximum is almost the same for both sets of model parameters. Because of the superconductivity, the location of the maximum is shifted to a higher temperature, $T_{\phi,\text{max}} \simeq T_{0,\text{max}}/\sqrt{\mathcal{N}}$, where \mathcal{N} is the same parameter that appears in Eq. (10). Taking the shift of the maximum into account, we find that the enhancement relative to the normal phase is observed for $T_* \leq T \leq T_c$ with $T_* \simeq T_{0,\text{max}}/\mathcal{N}^{1/4}$ being the point between $T_{0,\text{max}}$ and $T_{\phi,\text{max}}$, at which the bulk viscosities for the normal and superconducting phases cross. At lower temperatures, $T < T_*$, the effect of color superconductivity is opposite: it reduces the bulk viscosity.

The range of temperatures, in which the bulk viscosity increases relative to the normal phase of quark matter, depends on the value of the critical temperature T_c and the frequency of oscillations. While the actual enhancement of the viscosity also depends on the specific pattern of spin-one pairing, the qualitative features in all four phases studied here are similar. As an example, let us consider the CSL phase in more detail. In Fig. 3, we show the contour plot for the bulk viscosity enhancement factor due to color superconductivity. The ratio $\zeta_{\text{CSL}}/\zeta_{\text{normal}}$ is larger than 1 only in the colored regions in Fig. 3. In white regions, it is either 1 (when $T > T_c$) or less than 1 (otherwise).

As evident from Fig. 3, the enhancement of the bulk viscosity by spin-one color superconductivity occurs in a rather wide range of temperatures, especially when the frequency of density oscillations is not too large and the value of T_c is not too small. At $\tau^{-1} = 10$ Hz, for example, it extends over an order of magnitude or more in temperature, provided $T_c \gtrsim 100$ keV. At $\tau^{-1} = 1000$ Hz, in contrast, an order of magnitude or wider temperature range for the enhancement is seen only if $T_c \gtrsim 1$ MeV. (It should be noted that, in the case $\tau^{-1} = 10$ Hz shown

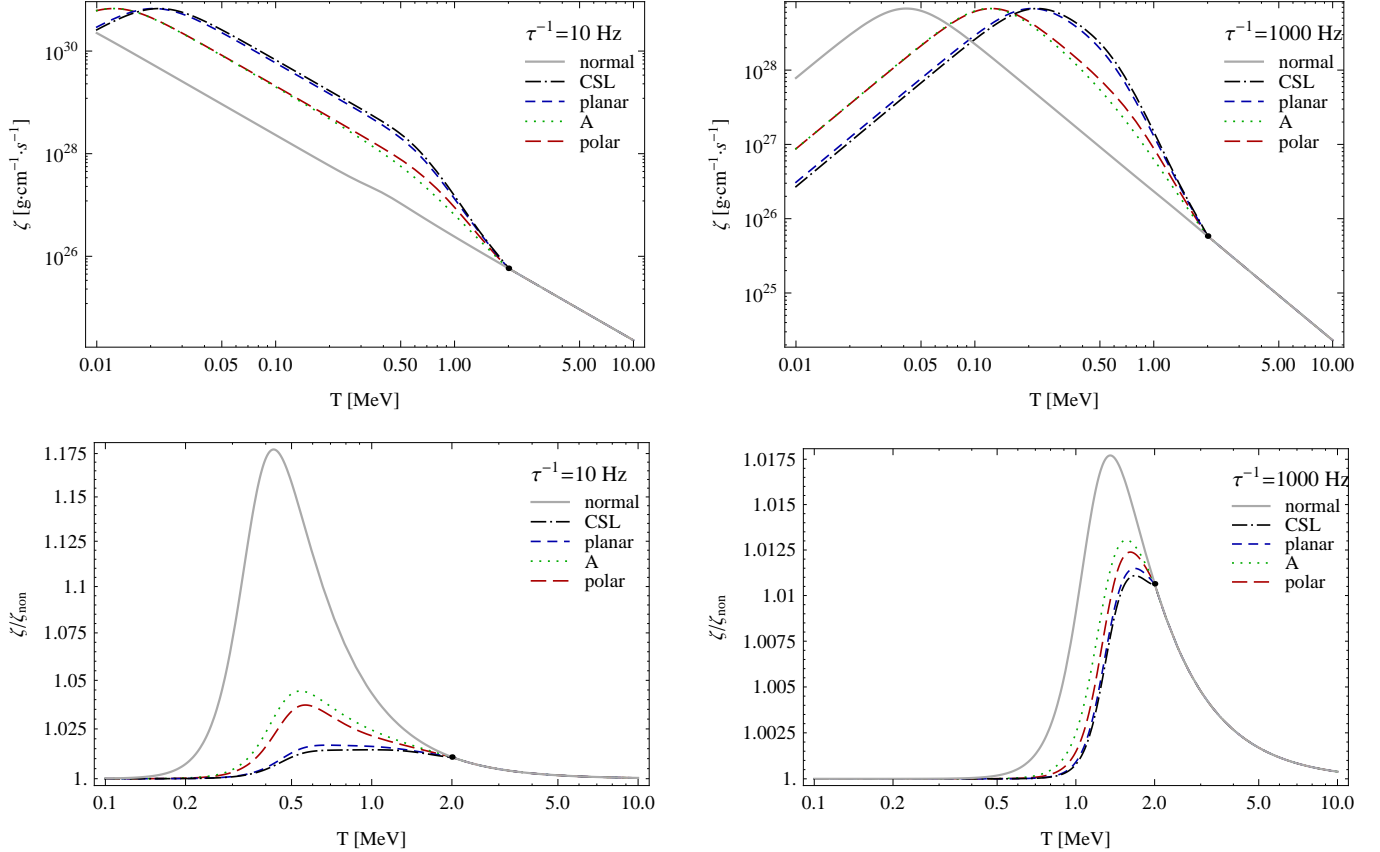


FIG. 4: (color online). Temperature dependence of bulk viscosity ζ and the ratio ζ/ζ_{non} for model parameters in Set B and the spin-one color superconducting critical temperature $T_c = 2$ MeV. The results for two fixed frequencies of the density oscillations are shown.

in the left panel in Fig. 3, the ratio $\zeta_{\text{CSL}}/\zeta_{\text{normal}}$ is truly less than 1 in a small white region just below the $T = T_c$ line. This “abnormality” is due to a subtle interplay between the semi-leptonic and nonleptonic processes when the value of T_c is fine-tuned to be near T_{hump} .)

By ignoring the subtle complications due to the semi-leptonic hump around T_{hump} , we find that the enhancement of the bulk viscosity in the window of temperatures $T_\star \leq T \leq T_c$ (as well as the suppression at lower temperatures, $T < T_\star$) is primarily due to the reduction of the nonleptonic rate λ_1 in color-superconducting phases. At temperatures below T_c , when all gapped quasiparticles effectively cease to contribute, the corresponding reduction factor for the rate is approximately given by the value of \mathcal{N} . This means that the enhancement factor for the viscosity approaches its inverse value, \mathcal{N}^{-1} . By making use of the numerical results for \mathcal{N} , we find that the enhancement factor for the bulk viscosity reaches up to about 9 in the A- and polar phases, 25 in the planar phase and 29 in the CSL phase. (The suppression factors at $T < T_\star$ approach the same values.) In the region of the hump, of course, the behavior is more complicated, but the overall effect of superconductivity is still mainly to increase the bulk viscosity.

The numerical results in the case of the model parameters in Set B are shown in Fig. 4. The qualitative features are similar to those obtained for Set A. However, the effect of the semi-leptonic processes is less pronounced: the corresponding hump is almost non-existent and the ratio ζ/ζ_{non} does not much deviate from 1. At the same time, the effect of color superconductivity is very well pronounced. Compared to the normal phase result, an enhancement of the bulk viscosity by a factor of about \mathcal{N}^{-1} is seen in a relatively wide window of temperatures from T_\star to T_c .

IV. DISCUSSION

In this study, we calculated the bulk viscosity in spin-one color-superconducting strange quark matter by carefully taking into account the interplay between the nonleptonic and semi-leptonic weak processes.

As expected, the nonleptonic processes give the dominate contribution to the viscosity in a wide range of parameters. Yet, as in the normal phase [34], the semi-leptonic processes may also lead to a substantial correction in a window of temperatures around T_{hump} , see

Eq. (16). The value of T_{hump} scales as $1/\tau^{1/4}$ and happens to be of order 1 MeV for millisecond pulsars. The size and the relative importance of the hump can be conveniently measured by the ratio ζ/ζ_{non} when it is noticeably larger than 1. For millisecond pulsars, however, this ratio remains close to 1. The effect is more pronounced when the period is a few orders of magnitude longer. We also find that the corresponding hump in the temperature dependence of the bulk viscosity of color superconductors is partially washed out compared to the normal phase. The higher is T_c relative to T_{hump} , the larger wash out of the hump is seen. At sufficiently low T_c , i.e., $T_c \lesssim T_{\text{hump}}$, the hump occurs in the normal phase and, therefore, its shape is unaffected by color superconductivity.

If the critical temperature of the spin-one color-superconducting phase transition T_c is larger than $T_{0,\text{max}}$, see Eq. (17), the main effect of color superconductivity is an overall increase of the bulk viscosity in a range of subcritical temperatures, $T_* \leq T \leq T_c$, see Fig. 3. The corresponding range of temperatures widens with increasing the value of T_c and with decreasing the frequency of oscillations. The increase of the viscosity is primarily due to the suppression of the nonleptonic rate by color superconductivity. At almost all temperatures below T_c , the rate is dominated by the ungapped quasiparticles, whose relative contribution is scaled by the factor \mathcal{N} with respect to the normal phase (note that $\mathcal{N} < 1$). It is the inverse value \mathcal{N}^{-1} that determines the maximal enhancement of the bulk viscosity at subcritical temperatures. The corresponding enhancement factor is equal to 9 in the A - and polar phases, about 25 in the planar phase and about 29 in the CSL phase. (At temperatures below T_* , color superconductivity leads to a suppression of the bulk viscosity, and the maximal suppression will approach the same value of \mathcal{N}^{-1} .)

In relation to this result, it might be appropriate to note that a similar enhancement mechanism was previously observed for spin-zero color superconductors [36]. A special feature of spin-one color superconductivity is that the maximum enhancement factor can be much larger.

In our analysis, we utilized the same spin-one pairing pattern as in Refs. [20–22]. In the case of zero quark masses, the main signature of the corresponding phases is the presence of ungapped quasiparticles. When quarks have small masses, the gaps of the corresponding modes are of order $\phi m/\mu$. These may be still too small to significantly affect our main results. However, if the spin-one gaps are larger, as some studies suggest [44], the suppression of the nonleptonic rates and, therefore, the enhancement of the bulk viscosity in color superconducting matter may turn out to be even stronger.

In application to compact stars, we may speculate that the transition to a spin-one color superconducting phase in a stellar core can have a stabilizing effect against the r -modes driven by the gravitational radiation [25]. If the critical temperature of the corresponding phase transition is on the order of or above 1 MeV, the correspond-

ing dynamics can affect even relatively young stars. The study of the actual quantitative effect that spin-one color superconductivity has on the reduction of the instability window in the pulsar frequency and temperature plane can be done along the lines of Refs. [45–47]. However, this problem is beyond the scope of the present paper.

Acknowledgments

The authors would like to thank Mark Alford, Thomas Schäfer, Andreas Schmitt and Kai Schwenzer for useful comments. The work of X.W. is also supported in part by the Arizona State University Graduate Fellowship. The work of I.A.S. is supported in part by the start-up funds from the Arizona State University and by the U.S. National Science Foundation under Grant No. PHY-0969844.

Appendix A: λ -rates of semi-leptonic (Urca) processes

The rates of the semi-leptonic processes in spin-one color superconducting quark matter were calculated in Ref. [37]. The general expression for the rate takes the following form:

$$\lambda_i = \lambda_i^{(0)} \left[\frac{1}{3} + \frac{2}{3} H \left(\frac{\phi}{T} \right) \right] \quad \text{for } i = 2, 3, \quad (\text{A1})$$

where $\lambda_i^{(0)}$ is the corresponding rate in the normal phase of quark matter and $H(\phi/T)$ is a phase-specific suppression factor. By construction, it satisfies the constraint $H(0) = 1$, which corresponds to the case of the normal phase. We used the numerical data of Ref. [37] to obtain the following fits for the suppression factors as functions of the dimensionless ratio $\varphi \equiv \phi/T$ in the four spin-one color superconducting phases of quark matter:

$$H^A(\varphi) = \frac{a_1\varphi^4 + b_1\varphi^3 + c_1\varphi^2 + d_1}{\varphi^5 + e_1\varphi^3 + f_1\varphi^2 + d_1}, \quad (\text{A2})$$

where $a_1 = 1.069$, $b_1 = -0.2187$, $c_1 = 3.666$, $d_1 = 21.50$, $e_1 = 1.333$ and $f_1 = 9.349$,

$$H^{\text{polar}}(\varphi) = \frac{a_2\varphi^3 + b_2\varphi^2 + c_2}{\varphi^5 + d_2\varphi^4 + e_2\varphi^3 + f_2\varphi^2 + c_2}, \quad (\text{A3})$$

where $a_2 = \pi$, $b_2 = 21.94$, $c_2 = 1386$, $d_2 = 6.994$, $e_2 = 11.20$ and $f_2 = 214.0$,

$$H^{\text{planar}}(\varphi) = \frac{a_3\varphi^{3.5} + b_3\varphi^3 + c_3\varphi^2 + d_3(1 + \varphi)}{\varphi^3 + e_3\varphi^2 + d_3} e^{-\varphi}, \quad (\text{A4})$$

where $a_3 = 0.917$, $b_3 = 0.456$, $c_3 = 11.69$, $d_3 = 34.0$ and $e_3 = 4.221$,

$$H^{\text{CSL}}(\varphi) = \frac{a_4\varphi^4 + b_4\varphi^3 + c_4\varphi^2 + d_4(1 + \sqrt{2}\varphi)}{\varphi^3 + e_4\varphi^2 + d_4} e^{-\sqrt{2}\varphi}, \quad (\text{A5})$$

where $a_4 = 1.034$, $b_4 = 1.001$, $c_4 = 9.735$, $d_4 = 13.81$ and $e_4 = 1.684$.

Appendix B: λ -rates of nonleptonic processes

The λ -rate of the nonleptonic processes in spin-one color superconducting quark matter was calculated in Ref. [40]. The general expression for the rate takes the following form:

$$\lambda_1 = \lambda_1^{(0)} \left[\mathcal{N} + (1 - \mathcal{N}) \tilde{H} \left(\frac{\phi}{T} \right) \right], \quad (\text{B1})$$

where $\lambda_1^{(0)}$ is the corresponding rate in the normal phase of quark matter, \mathcal{N} is a constant that determines the relative contribution of ungapped quasiparticles to the rate, and $\tilde{H}(\phi/T)$ is a phase-specific suppression factor due to gapped quasiparticles. The normal phase corresponds to $\phi = 0$, in which case there is no suppression and $\tilde{H}(0) = 1$. The value of \mathcal{N} for each phase reads

$$\mathcal{N}^A = \frac{1}{9}, \quad (\text{B2a})$$

$$\mathcal{N}^{\text{polar}} = \frac{1}{9}, \quad (\text{B2b})$$

$$\mathcal{N}^{\text{planar}} \approx 0.0393, \quad (\text{B2c})$$

$$\mathcal{N}^{\text{CSL}} = \frac{928}{27027}. \quad (\text{B2d})$$

For this study we used the numerical data of Ref. [40] to obtain the following fits for the suppression factors as functions of the dimensionless ratio $\varphi \equiv \phi/T$:

$$\tilde{H}^A(\varphi) = \frac{\alpha_1 \varphi^2 + \beta_1}{\varphi^3 + \gamma_1 \varphi^2 + \beta_1}, \quad (\text{B3})$$

where $\alpha_1 = 0.1247$, $\beta_1 = 12.60$ and $\gamma_1 = 5.042$,

$$\tilde{H}^{\text{polar}}(\varphi) = \frac{\alpha_2 \varphi^2 + \beta_2}{\varphi^4 + \gamma_2 \varphi^2 + \beta_2}, \quad (\text{B4})$$

where $\alpha_2 = 0.0271$, $\beta_2 = 65.45$ and $\gamma_2 = 13.35$,

$$\tilde{H}^{\text{planar}}(\varphi) = \frac{\alpha_3 \varphi^4 + \beta_3 \varphi^3 + \gamma_3 \varphi^2 + \delta_3(1 + \varphi)}{\varphi^2 + \delta_3} e^{-\varphi}, \quad (\text{B5})$$

where $\alpha_3 = 0.0717$, $\beta_3 = -0.2663$, $\gamma_3 = 1.108$ and $\delta_3 = 4.561$,

$$\tilde{H}^{\text{CSL}}(\varphi) = \frac{\alpha_4 \varphi^4 + \beta_4 \varphi^2 + \gamma_4(1 + \sqrt{2}\varphi)}{\varphi^3 + \delta_4 \varphi^2 + \gamma_4} e^{-\sqrt{2}\varphi}, \quad (\text{B6})$$

where $\alpha_4 = 0.6981$, $\beta_4 = -2.045$, $\gamma_4 = 4.482$ and $\delta_4 = -1.217$.

-
- [1] C. L. Fryer, *Astrophys. J.* **522** 413 (1999); K. Sumiyoshi, S. Yamada, and H. Suzuki, *Astrophys. J.* **667** 382 (2007).
 - [2] R. Oechslin and H.-T. Janka, *Phys. Rev. Lett.* **99**, 121102 (2007).
 - [3] A. W. Steiner, J. M. Lattimer and E. F. Brown, arXiv:1005.0811.
 - [4] F. Özel, G. Baym and T. Güver, arXiv:1002.3153 [astro-ph.HE].
 - [5] F. Özel, *Nature* **441**, 1115 (2006).
 - [6] M. Alford, D. Blaschke, A. Drago, T. Klahn, G. Pagliara and J. Schaffner-Bielich, *Nature* **445**, E7 (2007) [arXiv:astro-ph/0606524].
 - [7] D. Ivanenko and D. F. Kurdgelaidze, *Lett. Nuovo Cim.* **2**, 13 (1969); N. Itoh, *Prog. Theor. Phys.* **44**, 291 (1970); F. Iachello, W. D. Langer, and A. Lande, *Nucl. Phys. A* **219**, 612 (1974).
 - [8] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
 - [9] G. Zweig, preprint CERN-TH-401 (1964).
 - [10] J. C. Collins and M. J. Perry, *Phys. Rev. Lett.* **34**, 1353 (1975).
 - [11] H. D. Politzer, *Phys. Rev. Lett.* **30**, 1346 (1973); D. J. Gross and F. Wilczek, *Phys. Rev. D* **8**, 3633 (1973); *Phys. Rev. D* **9**, 980 (1974).
 - [12] B. C. Barrois, *Nucl. Phys. B* **129**, 390 (1977).
 - [13] D. Bailin and A. Love, *Phys. Lett. B* **137**, 348 (1984).
 - [14] K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333.
 - [15] M. G. Alford, *Ann. Rev. Nucl. Part. Sci.* **51** (2001) 131.
 - [16] S. Reddy, *Acta Phys. Polon. B* **33**, 4101 (2002).
 - [17] M. Buballa, *Phys. Rept.* **407**, 205 (2005).
 - [18] I. A. Shovkovy, *Found. Phys.* **35**, 1309 (2005).
 - [19] M. G. Alford, A. Schmitt, K. Rajagopal, and T. Schäfer, *Rev. Mod. Phys.* **80**, 1455 (2008).
 - [20] T. Schäfer, *Phys. Rev. D* **62**, 094007 (2000).
 - [21] A. Schmitt, Q. Wang and D. H. Rischke, *Phys. Rev. D* **66**, 114010 (2002); *Phys. Rev. Lett.* **91**, 242301 (2003).
 - [22] A. Schmitt, *Phys. Rev. D* **71**, 054016 (2005).
 - [23] M. G. Alford, J. A. Bowers, J. M. Cheyne, and G. A. Cowan, *Phys. Rev. D* **67**, 054018 (2003).
 - [24] N. Iwamoto, *Phys. Rev. Lett.* **44**, 1637 (1980).
 - [25] N. Andersson, *Astrophys. J.* **502**, 708 (1998).
 - [26] J. Madsen, *Phys. Rev. Lett.* **81**, 3311 (1998).
 - [27] Q. D. Wang and T. Lu, *Phys. Lett.* **148B**, 211 (1984).
 - [28] R. F. Sawyer, *Phys. Lett. B* **233**, 412 (1989).
 - [29] J. Madsen, *Phys. Rev. D* **46**, 3290 (1992).
 - [30] Z. Xiaoping, L. Xuewen, K. Miao, and Y. Shuhua, *Phys. Rev. C* **70**, 015803 (2004); Z. Xiaoping, K. Miao, L. Xuewen, and Y. Shuhua, *Phys. Rev. C* **72**, 025809 (2005).
 - [31] X.-p. Zheng, S.-h. Yang, and J.-R. Li, *Phys. Lett. B* **548**, 29 (2002).
 - [32] H. Dong, N. Su, and Q. Wang, *Phys. Rev. D* **75**, 074016 (2007).
 - [33] M. G. Alford, S. Mahmoodifar and K. Schwenzer, arXiv:1005.3769 [nucl-th].
 - [34] B. A. Sa'd, I. A. Shovkovy, and D. H. Rischke, *Phys.*

- Rev. D **75**, 125004 (2007).
- [35] M. G. Alford, M. Braby, S. Reddy, and T. Schäfer, Phys. Rev. C **75**, 055209 (2007); M. G. Alford, M. Braby, and A. Schmitt, J. Phys. G **35**, 115007 (2008).
 - [36] M. G. Alford and A. Schmitt, J. Phys. G **34**, 67 (2007).
 - [37] B. A. Sa'd, I. A. Shovkovy, and D. H. Rischke, Phys. Rev. D **75**, 065016 (2007).
 - [38] B. A. Sa'd, arXiv:0806.3359 [astro-ph].
 - [39] M. Mannarelli and C. Manuel, Phys. Rev. D **81**, 043002 (2010).
 - [40] X. Wang, H. Malekzadeh and I. A. Shovkovy, Phys. Rev. D **81**, 045021 (2010).
 - [41] J. Madsen, Phys. Rev. D **47**, 325 (1993).
 - [42] J.D. Anand, A. Goyal, V.K. Gupta, and S. Singh, Astrophys. J. **481**, 954 (1997).
 - [43] J. R. Ellis, E. Gardi, M. Karliner and M. A. Samuel, Phys. Rev. D **54**, 6986 (1996).
 - [44] F. Marhauser, D. Nickel, M. Buballa and J. Wambach, Phys. Rev. D **75**, 054022 (2007).
 - [45] J. Madsen, Phys. Rev. Lett. **85**, 10 (2000).
 - [46] N. Andersson, D. I. Jones and K. D. Kokkotas, Mon. Not. Roy. Astron. Soc. **337**, 1224 (2002).
 - [47] P. Jaikumar, G. Rupak and A. W. Steiner, Phys. Rev. D **78**, 123007 (2008).